**Reliability Models**

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# 1. Reliability and Failure Rate Functions

Let be the random variable defining the lifetime of the component with distribution function , which is the time the component will operate before failure. The cumulative distribution function (**CDF**) of the random variable is given by:

If is a differentiable function, then the probability density function (**PDF**) of is given by:

The **reliability function** of the component is given by:

It is the probability that the component will operate after time , sometimes called survival probability. The failure rate of a system during the interval is the probability that a failure per unit time occurs in the interval, given that a failure has not occurred prior to , the beginning of the interval. The **failure rate function (i.e. instantaneous failure rate, conditional failure rate)** of hazard function is defined as the limit of the failure rate as the interval approaches zero. Hence

The failure rate function is the rate of change of the conditional probability of failure at time .

Generally is the one tabulated because it is measured experimentally and because it tends to vary less rapidly with time than the other parameters. When is given, all other three paramters can be computed as follows:

It measures the likelihood that a component that has operated up until time fails in the next instant of time. The mean time between failure (**MTBF**) can be obtained by finding the expected value of the random variable , time to failure. Hence

# 2. Commonly Used Distributions

## a. The Binomial Distribution

The binomial distribution is a discrete distribution. It arises in cases where many independent trials can result in either a success or a failure and we are interested in finding the probability of having successes in such trials, and its expression is given by:

The mean and variance of the binomial distribution are given by:

## b. The Poisson Distribution

The Poisson distribution is also a discrete distribution. It can be used when one is interested in finding the probability of having failures during a certain period of interest, and its expression is given by:

The mean and variance of the Poisson distribution are given by:

## c. The Normal Distribution

The normal distribution is a continuous distribution. It takes the well known bell shape and is symmetrical about its mean value, and its probability density function is given by:

And its cumulative distribution is given by

The mean and variance of the Poisson distribution are given by:

The failure rate function, , corresponding to a normal distribution is a monotonically increasing function of .

## d. The Lognormal Distribution

The probability density function of the lognormal is given by

Note that if a random variable is defined as , where is lognormally distributed with parameters and , then is normally distributed with mean and standard deviation . The mean and variance of the lognormal are given by:

## e. The Exponential Distribution

The exponential distribution can be used in reliability as a model of the time to failure of a component, and its probability density function is given by

The mean and variance of the exponential distribution are given by

And its cumulative distribution is given by:

## f. The Weibull Distribution

The probability density function of the three-parameter Weibull distribution is given by:

Where , and is the scale parameter, is the shape parameter, and is the location parameter.

## g. The Erlangian Distribution

The Erlangian distribution is a special case of Gamma distribution, with shape parameter an integer. Note that this restriction is not enforced. It will, however, generate a warning the first time a non-integer value is used for the shape parameter. Its probability density function is given by:

## h. The Gamma Distribution

The gamma failure probability density obeys the equation

Where is the shape parameter, and is the rate parameter (an inverse scale parameter)

## i. The Fatigue Life Distribution (Birnbaum-Saunders)

The fatigue life failure probability density obeys the equation

## j. The Exponentiated Weibull Distribution

The exponentiated Weibull failure probability density obeys the equation

## k. The Bathtub Distribution

For , and . The constants and are shape parameters and the constants and are scale parameters. For this distribution:

And

## l. The Power Law Model for Failure Rate Function

The hazard rate satisfies a power law as a function of time:

Where and .

## m. The Log Linear Model for Failure Rate Function

The hazard rate satisfies a exponential law as a function of time:

Where and .

Scipy.stats (<https://docs.scipy.org/doc/scipy/reference/stats.html>) can be used to generate functions including .

# 3. Reliability User Cases:

Case 1:

**Input**: single value generated via RAVEN sampler

a. hyper parameters, i.e. , can be constant or given distribution (RAVEN sampled variables)

b. mission time, i.e. , can be constant or given distribution (raven sampled variables)

**Output**: return single values of

Case 2:

**Input**:

single value generated via RAVEN sampler

a. hyper parameters, i.e. , can be constant or given distribution (RAVEN sampled variables)

time-series (vector or array) generated by RAVEN external models

b. mission time, i.e. , can be constant or given distribution (raven sampled variables)

**Output**: return time-series of at different

Case 3:

**Input**:

Single value generated via RAVEN Sampler Functions

Where become the hyper parameters that can be sampled via RAVEN

a. hyper parameters, i.e. , can be constant or given distribution (RAVEN sampled variables)

b. mission time, i.e. , can be constant or given distribution (raven sampled variables)

**Output**: return single values of

Case 4:

**Input**:

Time-series values generated via RAVEN External Models

Where become the hyper parameters that can be sampled via RAVEN

a. hyper parameters, i.e. , can be constant or given distribution (RAVEN sampled variables)

time-series (vector or array) generated by RAVEN external models

b. mission time, i.e. , can be constant or given distribution (raven sampled variables)

Ensemble Model is used to link external models with reliability models

**Output**: return time-series of at different

Case 5 (???):

**Input**:

Time-series values generated via RAVEN External Models with stochastics (i.e. Gaussian Process model)

Where become the hyper parameters that can be sampled via RAVEN

a. hyper parameters, i.e. , can be constant or given distribution (RAVEN sampled variables)

time-series (vector or array) generated by RAVEN external models

b. mission time, i.e. , can be constant or given distribution (raven sampled variables)

Ensemble Model is used to link external models with reliability models

**Output**: return time-series of at different

Question: How to handle intrinsic stochastics

# 4. Procedures for Bayesian Updates:

1. determine prior distribution for input parameters

2. determine the likelihood functions

Assume is a vector of model parameters and is a vector of observable quantities, or data. The simulation model yields predictions of data as a function of the parameters . In the Bayesian setting, both and are random variables. We use Bayes’s rule to define a posterior probability density for the model parameters , given an observation of the data :

Data thus enters the formulation through the likelihood , which may be viewed as a function of : . A simple model for the likelihood assumes that independent additive errors account for the deviation between predicted and observed values of :

where components of are i.i.d. random variables with density . A typical assumption is , in which case becomes . The likelihood is thus

In this simple model, may encompass both measurement error (e.g. sensor noise) and model error. Note that in situations when one believes the measurement data are correlated, needs to be replaced by an error covariance matrix . It is expected that the measurement error variances are provided along with benchmark data.

# 5. RAVEN Bayesian Updates Design

a. Process of PyMC:

1. Define prior distribution for hyper-parameters
2. Define likelihood functions with observations
3. Choose sampling strategies to obtain posterior distribution for each hyper-parameter

b. Process of RAVEN

1. Employ Distributions to define prior distribution and Samplers from RAVEN to convert RAVEN Distribution entities into PyMC prior distributions
2. Construct a new model entity, i.e. likelihood models as shown in section 4, to handle both experimental data and simulation results
3. Employ SingleRun step (encapsulate PyMC mcmc samplers) to obtain the posterior distributions.
4. How to sample posterior distribution? Brute-force, resampling, or construction PDF using KDE? Or extend the custom distribution to construction distributions from data object?
5. KDE or other methods can be used to restrict posterior distribution
6. Employ posterior distributions for further analysis (employed in another ensemble model for optimizations)

# 6. SR2ML Bayesian Updates Design

Bayesian update will be handled by the reliability external models.

Prior distribution 🡪 likelihood function 🡪 sampling strategies 🡪 pm.sample\_posterior\_predictive